12.5 Equations of lines and planes in space (DRILL)

CONCEPTS

To find the equation of a line, you need

[a] a point on the line, and

[b] a direction vector that the line is parallel to

The direction vector can either be

[i] given explicitly,

[ii] found by using a vector through 2 points on the line
(the line's direction vector will be parallel to that vector),

[iii] given implicitly from a line that the line is parallel to
(the lines' direction vectors will be parallel),

[iv] given implicitly from a plane that the line is perpendicular to
(the line's direction vector will be parallel to the plane's normal vector), or

[v] found when given 2 planes that the line is parallel to
(the line's direction vector will be perpendicular to both planes' normal vectors)

To find the equation of a plane, you need

[a] a point on the plane, and

[b] a normal vector that the plane is perpendicular to

The normal vector can either be

[i] given explicitly,

[ii] found by using 2 vectors through 3 points on the plane
(the plane's normal vector will be perpendicular to both vectors),

[iii] given implicitly from a plane that the plane is parallel to
(the planes' normal vectors will be parallel),

[iv] given implicitly from a line that the plane is perpendicular to
(the plane's normal vector will be parallel to the line's direction vector),

[v] found when given 2 lines that the plane is parallel to
(the plane's normal vector will be perpendicular to both lines' direction vectors), or

[vi] found when given 2 planes that the plane is perpendicular to
(the plane's normal vector will be perpendicular to both planes' normal vectors)

PROBLEMS

Print out this list of questions.

Cut the print out apart so each question is on a separate slip of paper.

Select the slips of paper in random order and solve them.

Try to solve the problems without repeatedly looking back at the previous page.
Instead, ask yourself "How should the direction vector(s) and/or normal vector(s) be related ?".

Find parametric AND symmetric equations of the following lines:

[1] passess through $(1, 4, 2)$, parallel to $3\mathbf{j} + 2\mathbf{k} - \mathbf{i}$

[2] passess through $(-2, 1, -4)$ and $(1, 4, 2)$

[3] passes through $(5, -3, -2)$, parallel to line $z = 7 - 6t, \quad y = 2t + 4, \quad x = 3t - 2$

[4] passes through $(-1, -5, 3)$, parallel to line $\frac{y+5}{7} = \frac{2-x}{5} = \frac{z-4}{3}$

[5] passes through $(6, -8, 2)$, parallel to planes $2x + 3y - z = 4$ and $2z + 3x - y = 4$

[6] passes through $(-5, 7, 6)$, perpendicular to plane $2x + 3z - y = 4$

Find the standard (point-normal) AND general equations of the following planes:

[7] passess through $(1, 4, 2)$, perpendicular to $3\mathbf{i} + 2\mathbf{k} - \mathbf{j}$

[8] passess through $(-2, 1, -4)$, $(1, 4, 2)$ and $(3, 0, -3)$

[9] passes through $(5, -3, -2)$, parallel to plane $2z + 3x - y = 4$

[10] passes through $(-1, -5, 3)$, perpendicular to line $\frac{x+5}{7} = \frac{2-y}{5} = \frac{z-4}{3}$

[11] passes through $(6, -8, 2)$, parallel to lines $y = 7 - 6t, \quad x = 2t + 4, \quad z = 3t - 2$ and $\frac{z+5}{7} = \frac{2-y}{5} = \frac{x-4}{3}$

[12] passes through $(6, -8, 2)$, perpendicular to planes $2y + 3z - x = 4$ and $2y + 3x - z = 4$

ANSWERS

Answers are in random order (answers in the same row are not necessarily for the same question)

$x = 5 + 3t, \quad y = -3 + 2t, \quad z = -2 - 6t$	$1 - x = \frac{y-4}{3} = \frac{z-2}{2}$
$x = -5 + 2t, \quad y = 7 - t, \quad z = 6 + 3t$	$\frac{-x-1}{5} = \frac{y+5}{7} = \frac{z-3}{3}$
$x = 1 - t, \quad y = 4 + 3t, \quad z = 2 + 2t$	$\frac{x-5}{3} = \frac{y+3}{2} = \frac{-z-2}{6}$
$x = 6 + 5t, \quad y = -8 - 7t, \quad z = 2 - 11t$	$\frac{x+2}{3} = \frac{y-1}{3} = \frac{z+4}{6}$ OR $x + 2 = y - 1 = \frac{z+4}{2}$
$x = -1 - 5t, \quad y = -5 + 7t, \quad z = 3 + 3t$	$\frac{x-6}{5} = \frac{-y-8}{7} = \frac{2-z}{11}$
$x = -2 + 3t, \quad y = 1 + 3t, \quad z = -4 + 6t$	$\frac{x+5}{2} = 7 - y = \frac{z-6}{3}$
$3(x-1) - (y-4) + 2(z-2) = 0$	$-27x - 5y + 8z + 106 = 0$
$9(x+2) + 27(y-1) - 18(z+4) = 0$ OR $(x+2) + 3(y-1) - 2(z+4) = 0$	$x + 3y - 2z - 9 = 0$
$3(x-5) - (y+3) + 2(z+2) = 0$	$x - y + z - 16 = 0$
$7(x+1) - 5(y+5) + 3(z-3) = 0$	$3x - y + 2z - 3 = 0$
$-27(x-6) - 5(y+8) + 8(z-2) = 0$	$3x - y + 2z - 14 = 0$
$-8(x-6) + 8(y+8) - 8(z-2) = 0$ OR $(x-6) - (y+8) + (z-2) = 0$	$7x - 5y + 3z - 27 = 0$